# DR. BABASAHEB AMBEDKAR MARATHWADA UNIVERSITY



# **DEPARTMENT OF MATHEMATICS**

# **SYLLABUS**

FOR P. G. - C.E.T.

M.Sc.- Mathematics and M.Sc.- Applied Mathematics

For the Academic Year

2019-2020

#### Differential Calculus

# 1. Prerequisite:

Functions: Domain and range of a function, independent and dependent variables, functions and rational functions, constant functions and identity functions, one-one functions, onto function, invertible functions, composite function.

Limit and Continuity: Limit of a function, left handed and right handed limits, nonexistence of limit, theorems on limits (statements only), theorems on continuity Segments only. Discontinuity, types of discontinuity

#### 2. Differentiations:

Derivative of a functions, derived function, derivability implying continuity, geometrical interpretation of a derivative, hyperbolic functions, derivatives of hyperbolic and inverse hyperbolic functions, logarithmic differentiation derivative of implicit functions.

#### 3. Successive Differentiation:

Higher order derivatives calculation of n th derivative of rational functions, the n th derivatives of the products of the powers of since and consigns, Leibnitz's theorem n th derivative of the product of two functions.

#### 4. Mean Value Theorems:

Rolle's Theorem, Lagrange's mean value theorem, Meaning of the sign of the derivative, Cauchy's mean value theorem, higher derivatives, Taylor's theorem, Maclaurin's theorem, Maclaurin's power series for a given function.

### 5. Partial Differentiation:

Function of two variables, limit of a function of two variables, continuity of a function of two variables at a point, limit of a continuous function, partial derivatives, partial derivatives of higher order, homogeneous function, Euler's theorem on homogeneous function, total differentials, differentiation of composite function and implicit function.

# 6. Prerequisite:

Scalar product of two vectors, sign of the scalar product, length of a vector as a scalar product, angle between two vectors, commutativity, distributivity, right handed and left handed vector triads, vector product, some properties of vector product, scalar triple product, distributive law, some properties of scalar triple product, vector triple product.

# 7. Differential Operators:

Point Functions: Scalar valued point functions, vector valued point functions, limits and continuity, directional derivatives, Cartesian representation of point functions and their

directional derivatives, directional derivatives of point functions along co-ordinate axes and along any line, gradient of a scalar point function, character of gradient as a point function, the operator  $\nabla$ , operator a.  $\nabla$ , divergence and curl, gradient, divergence and curl of sums and product.

#### **Differential Equations**

# 1. Prerequisite:

Ordinary and Partial Differential Equations, order and degree of Differential Equations Solutions general, Particular, Singular.

2. Equations the First Order and of the Degree:

Exact Differential Equations, Linear equations, Equations reducible to the linear form

3. Linear Equations with Constant Coefficients:

Linear Equations, complementary functions, particular integral, complete integral, The linear equations with constant coefficients and second member zero, case of auxiliary equation having equal roots, case of auxiliary equation having imaginary roots. The symbol D, the linear equation with constant coefficients and second member a function of x, the symbolic function 1/f(D), methods of finding the particular integral, short methods of finding particular integrals corresponding to the terms  $e^{ax}$ ,  $x^m$ , sinax, cosax,  $e^{ax}V$  and xV in the second member.

4. Linear Equations with Variable Coefficients:

The homogenous linear equations, methods of finding solution, the symbolic functions f(D) and 1/f(D), methods of finding the particular integral, integral corresponding to a term of form  $x^m$  in the second member, equations reducible to homogenous linear form.

5. Exact Differential Equations and Equations of Particular Forms:

Exact differential equations, criterion of the form  $\frac{d^n y}{dx^n} = f(x)$ , equations of the form

$$\frac{d^2y}{dx^2} = f(xy).$$

6. Ordinary Differential Equations with more than two variables:

Simultaneous differential equations which are linear, simultaneous equations of the First order.

7. Partial Differential Equations:

Definitions derivation of a partial differential equation by the elimination of constants. Derivation of a partial differential equation by the elimination of arbitrary functions.

**Number Theory** 

### 1. Divisibility Theory in the Integers:

The Division Algorithm, The greatest common division, The Euclidean algorithm the Diophantine equations ax + by = c.

#### 2. Primes and their Distributions:

The Fundamental Theorem of Arithmetic.

# 3. The Theory of congruences:

Basic Properties of congruence's, linear congruence's.

#### 4. Fermat's Theorem:

Fermat's Factorization Theorem, The little Theorem, Wilson's Theorem.

#### 5. Number-Theoretic Functions:

The functions  $\tau$  and  $\sigma$ . The Mobius inversion formula.

# 6. Euler's Generalization of Fermat's Theorem:

Euler's Phi function, Euler's Theorem, Some properties of Phi function.

#### **Integral Transforms**

#### 1. Beta and Gamma Functions:

Euler's Integrals- Beta and Gamma functions, Elementary properties of Gamma Function, Transformation of Gamma Function, Another form of Beta Function, Relation between beta and Gamma functions, Other Transformations.

# 2. Laplace Transform:

Piece-wise or section continuity, function of exponential order, function of Class *A*, The transform concept, Laplace Transform, Notation. Some Standard results.

# 3. Inverse Laplace Transform:

Definition, Null function, Uniqueness of inverse Laplace transform, partial fractions, Heaviside's expansion formula, the complex inversion formula

## 4. Application to Differential Equations:

Differential Equation, Notations (Problems related to Ordinary Differential Equations only)

#### **Numerical Methods**

# 1. Solution of Algebraic and Transcendental Equations:

Introduction, Bisection method, Method of false position, Newton-Raphson method, Generalized Newton's method.

# 2. Interpolation:

Introduction, Finite differences. Forward differences, Backward differences, Central differences, Symbolic relations and separation of symbols, Differences of a polynomial, Newton's formula, Hermit's interpolation formula, Divided differences and their properties, Newton's general interpolation formula.

#### Partial Differential Equations

# 1. Prerequisites:

Derivation of a Partial Differential Equations by the elimination of arbitrary constants, Derivation of a Partial Differential Equation by the elimination of arbitrary functions.

#### 2. Partial Differential Equations of Order one (Linear Equations):

Definition of Partial Differential Equations, Lagrange's Linear Partial Differential Equation, Geometrical interpretation of the Lagrange's Linear Partial Differential Equations Pp+Qq =R.

# 3. Non-linear Partial Differential Equations of Order One:

Complete and Particular Integrals, General Integral, Singular Integra, Special method, Standard form I, Standard form II, Standard form III, Standard form IV, Charpit's method, Non-linear Partial Differential Equations of order one with three of more independent variables, Jacobi's method.

## 4. Linear Partial Differential Equations:

Definitions, Linear Homogeneous Partial Differential Equations with constant coefficients, Non-Homogeneous Linear Partial Differential Equations, equations reducible to Linear form with constant coefficients.

#### 5. Partial Differential Equations of Second Order:

Equations that can be integrated by inspection, Monge's method to solve the equation Rr+Ss+Tt=V, Method of Transformations (Canonical Forms)

# Real Analysis-I

# 1. Prerequisite:

Sets and elements, Operations on sets.

#### 2. Functions:

Functions, Real-Valued functions, Equivalence, Countability, Real numbers, Least Upper Bounds

# 3. Sequences of Real Numbers:

Definition of sequence and subsequence, Limit of a sequence, Convergent sequences, Divergent sequences, Bounded sequences, Monotone sequences, Operations on convergent sequences, Operations on divergent sequences, limit superior an limit inferior, Cauchy sequences

#### 4. Series of Real Numbers:

Convergence an divergence, Series with non-negative terms, Alternating series, Conditional convergence and convergence, Test for absolute convergence

#### 5. Jacobians:

Definitions, Case of function of functions, Jacobian of implicit functions, Necessary and sufficient condition for a Jacobian to vanish.

# Abstract Algebra-I

# 1. Prerequisite:

Sets Functions, Integers.

# 2. Group Theory:

Definition of a group, Some examples of groups, Some preliminary lemmas Subgroups, A counting Principle, Normal subgroups and quotient groups, Homomorphism, Automorphism.

# 3. Ring Theory:

Definition and examples of rings some special classes of ring, Ideals and quotient rings, more about ideals and quotient rings, Polynomial ring.

# Real Analysis- II

# 1. Limits in Metric Spaces:

Metric Spaces, Limits in metric spaces.

# 2. Continuous Functions of Metric Spaces:

Functions continuous on metric spaces, open sets, Closed sets.

# 3. Connectedness, Completeness and Compactness:

More about open sets, connected sets, bounded sets and totally bounded sets, complete metric spaces, Compact metric spaces, Continuous functions on compact metric spaces, Uniform continuity.

#### 4. Calculus:

Sets of measure zero, Definition of Riemann Integra, Existence of Riemann Integral, Fundamental Theorem of Calculus.

# 5. Fourier Series:

Introduction.

# Abstract Algebra-II

# 1. Vector Spaces and Modules:

Elementary basic concepts, Linear independence and bases, Dual spaces, linear product spaces, Modules.